STATISTICAL APPROACH IN DEVELOPING MATHEMATICAL MODELS FOR EVALUATING INTERNAL IRRADIATION

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Abstract

It is established that calculation of radioactive intake (or accumulation) into the body according to the controlled dosimetric parameter requires some statistic data about specific peculiarities of the investigated object.

This paper presents data about statistical distribution functions of the radionuclide concentration ratio at the sampling point to its concentration at the respiration zone, protective efficiency of individual protective means, elimination of the radionuclide from the body and others; it gives en estimate of the errors for determining an inhalation intake of radioactivity into the body.

Various techniques for controlling an internal radiation exposure is analysed.

1. Basic environmental data

In order to estimate accumulation of radioactive substances within the body of people by data on the contaminated air it is necessary to know aerosol characteristics averaged over human locations during the working shift with due account for the efficiency of individual protective means. But the necessary qualitative and quantitative data are usually fully lost when stationary samplers are used. This is confirmed by the tabulated data. The data obtained from stationary aerosol radiometers approach those from individual air samplers for rooms which lack local powerful sources of air contamination (Table 1). Otherwise discrepancy of the data will be by the order of several tens.

Table 1*

Comparative estimate of the data obtained from stationary and individual samplers

Characteristics of the air contaminating sources	The ratio of the data obtained from an individual sampler to those from a stationary one (the log-normal concentration distribution law)			
	mean	maximum (seen de facto)	median	standard geometric deviation β
Local sources of initial contamination are available	24	500	12	3,2
No sources of initial contamination	2.5	5	1.8	2.3

*Both for this Table, and for Table 2-4 there were usual statistical methods to estimate the statistical confidence of data.

Efficiency of individual respiratory protective means is of great importance for calculating the value of inhaled radioactivity.

Table 2 shows the practical decrease rate in contamination of the inhaled air by a respirator of the "Lepestok" type. Both the relatively small efficiency of respirators for low contaminated air and the besser efficiency (comparing to the value founded in the laboratory) for the higher level of air contamination are resulted from time of using of respirators during one working day. ("Lepestok" is the respirator of the simple type).

Table 2

Efficiency of the respirator "Lepestok"

The lewel of air contamination	The decrease rate in contamination of the inhaled air (follows the log-normal law)					
	in real conditions				in laboratory experiments (according to S. M. Gorodinsky)	
	average	maximum	median	standard geometrical deviation	average	median
Below APC	24	300	12	3.2	680	450
Above APC	350	2000	160	3.6		

Therefore both the stationary samplers and individual ones have some positive or negative aspects, but can't used in order to estimate the real individual intakes of airborn radioactive substances. This conclusion will be also confirmed by next discussion.

Dispersity of alpha-active airborn particles deposited at the external nosal orifices and present in the inhaled air and in the air of working premisis is estimated in Table 3, where the following designations are accepted:

r_g - geometrical mean radius,

r_{max} - maximum radius of the airborn particle in the sample.

Geometrical mean radius of the log-normal distribution is determined by the ratio

$$\ln r_g = \frac{1}{\sqrt{2\pi}\ln\beta}\int \ln r \cdot \exp\left\{-\left(\frac{\ln r/r_g}{\sqrt{2}\ln\beta}\right)^2\right\} d(\ln r) = \frac{1}{n}\sum_{i=1}^{n}\ln r_i ;$$

$\ln\beta$ - standard deviation of the radius logarithm;

$r_{ag} = r_g \cdot \beta^{0.5\ln\beta}$ - geometrical mean activity radius for the non-limited log-normal distribution.

$r_{ag(lim)}$ - geometrical mean activity radius for the limited log-normal distribution

$$r_{ag(lim)} = r_{ag}\cdot\beta^{-\left\{\frac{\exp\left[-\left(\frac{\ln r_{max}}{\sqrt{2}\ln\beta}\right)^2\right]}{0.5\sqrt{2\pi}\left[1+\Phi(\xi'_{r_{max}})\right]}\right\}} \quad (1), \qquad r_{am(lim)} = r_{ag}\beta^{\varphi_1^{-1}} \quad (2)$$

where

$$\xi'_{r_{max}} = \frac{1}{\lg\beta}\left[\lg r_{max} - (\lg r_g + 2.3\lg^2\beta)\right], \qquad \varphi_1 = 0.5\left[\Phi(\xi'_{r_{max}}) - 1\right]$$

$$\Phi(\xi_i) = \frac{2}{\sqrt{2\pi}}\int_0^{\xi_i} e^{-\xi^2/2}\,d\xi$$ - Gauss probability integral

$r_{am(lim)}$ - activity median radius with account for the limited log-normal distribution

Estimation of the confident interval indicated that with the level of significance 0.05 the intervals were not overlapped. This resalt is caused by the difference in the size distribution of airborn particles collected by three various techniques. Neither Fisher relation is followed. Therefore the difference of the mean standard deviations should be considered significant.

Table 3

Dispersity of airborn particles

Collection technique	Averaged dispersity of airborn particles					
	β	r_{g}, μm	r_{max}, μm	r_{ag}, μm	r_{ag} (lim) μm	r_{am} (lim) μm
Stationary sampler	2.4	3.8	44-120	39	20-32	20-39
Individual sampler	3.2	1.3	21-34	70	11-17	12-19
Smears from the nose	4.3	0.53	15-32	290	17-20	18-20

On the other hand limited aerodynamic activity radiuses with permissible dosimetric error coincide (Table 3). Therefore the activity fraction settled at various parts of the respiratory tract calculated by AMAD[1] will be equal. Individual protective respiratory devices are thought practically not to modify dispersity of the inhaled dust in the case of coarse-grained particles. Table 4 gives comparative results obtained by three various techniques in the course analysis of the daily inhalation intake for 3 groups of workers.

Table 4

Comparison of various techniques for determining the daily inhalation intake A by the average values in terms of A_{11} for the use of individual protective means

No i	Thechnique for determining the intake	The group of workers 1		2		3		The number of analyses for all groups	Note
		A_{1i}	β_{1i}	A_{2i}	β_{2i}	A_{3i}	β_{3i}		
1	Analysis of excretions	1	5.9	0.97	4.8	0.63	5.2	131	With account for solubility of dust determined by experiment
2	Smears from the nose	0.83	2.8	1.1	3.7	6	5.3	77	With account for dispersity of dust determined by stationary samplers
3	Individual samplers (without account for IPM)	160	2.4	450	4.3	790	3.2	94	Assuming the volume of the inhaled air/10 m^3

β_{ji} - corresponding A_{ji} distribution standard geometrical deviation.

Therefore it is believed reasonable that the value of individual inhalation intake (or accumulation) into the body should be calculated ether by the indications of smears taken from the external nosal orifices or by the excretion dates, whereas activity distribution over the respiratory tract may be estimated with due accuracy by means of stationary or individual samplers (if dispersity is correctly averaged by stationary samplers). The last remark is important for those working rooms where dispersity of airborn particles is changed as a function of working locations and the type of technological operation and consequently the level of dust penetrating the lungs is also changed.

Thus, determination of actual accumulation of activity within the body due to inhalation requires both measurement of individual inhalation intake and one of dispersity of the inhaled aerosols by a direct method. Determination of dispersity can be substituted by the estimation of the relative value of penetrating airborn particle fraction. Practically the most suitable is combination of the method for determining the intake by the smears from the nose and selective individual samplers provided with presettler.

If these requirements are not fullield the use of average values may lead to errors in calculating the individual intake by several orders of magnitude:

- up to 20 times due to disparity of stationary and individual sampling;
- up to 10 times due to errors in determining dispersity of the inhaled dust and up to 10 ÷ 100 times due to the differences in real effectivity.

Approximate lognormal space-time distribution statistics for radioactive concentration in the air of working premisis is currently given much consideration in literature[2]. Due to logarithmically normal fluctuations in the protective effectivity of individual protective means and due to generally random time schedule of work the above stated factors even after averaging over long time periods (calendar year) lead to the actual radioactive intake to individuals from the homogenous group being described by the lognormal distribution with the significant standard geometrical deviation (β). We found that β in this case can amount to 2.5-7. It should be emphasized that the groups should be clearly enough classified according to the radiation situation (by profession, by location and time, etc.). Otherwise the standard geometrical deviation will be much increased (up to 10 and more).

2. Basic human data

We belive that the lognormal distribution law of radioactivity eliminated from the body with urine and feci is an important half-empirical consideration (Table 4). This law is determined by both the lognormal intake pattern and the statistical character of metabolism within the body.

It will be reasonable to consider the statistical similitude principle as a general assumption for describing metabolism of radioactive substances within the body:

- the ratio X_i (radiation burden within the body or in some part of the body to a single intake "i" physiological cycles after intake) is a random value not depending on the intake value.

This makes it possible in many cases to describe the result of the i-th cycle in the form of the proportional effect law:

$$x_i - x_{i-1} \equiv \varphi_i x_{i-1} \qquad (3)$$

Here - a random function, if to apply to it reasonably general limitations and to use the central probability concept ultimate theorem it is possible to obtain an expression for the large values of "i" in the form of the lognormal distribution law of the value x_i. E. g. if to describe elimination of the substance from the body organ one may obtain:

$$P\left\{\frac{\ln x_i - \ln(x_0 e^{-\lambda i})}{\sqrt{i}\,\sigma} < y\right\} \underset{i\to\infty}{=} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{v^2}{2}} dv \qquad (4)$$

Were P - probability of inequality taken in brackets;
x_0 - the initial radioactive quantity in the organ;
$\lambda = -\ln(1+\varphi)$ - if to assume that φ is not dependent on x but is a random value with dispersion $\sigma(\sigma)$. With other initial conditions one may obtain a different expression for the lognormal distribution median (4) which in the general form is expressed as a function of the number of physiological cycles $f(i)$. It is difficult to obtain and consider the function $f(i)$ in the general form. Therefore currently it is reasonable to use the empirical fact on the lognormal distribution law of x_i values around $f(i)$ and a constant value of the standard geometrical deviation of the same individuum.

For example we found that 5-10 days after removal of the workers from the "hot" laboratory the ratio of the daily elimination of Po-210 for the previous day to the daily elimination for the next day fluctuates around the median value $\approx$ 1.0 following the lognormal distribution with the standard geometrical deviation $\approx$ 2.7. This agrees with the standard geometrical deviation of the primary value x_i $\beta = 2.0$.

Practically the same standard geometrical deviation is probably obtained when individual fluctuations of Pu-239 elimination from the body are estimated. W. S. Snyder[3] found that in 60% of cases of the plutonium daily elimination the difference from the "individual" curve was not more than two-fold and in 80% of cases it was not more than three-fold.

In addition to these metabolic data for polonium and plutonium one cite the experimental results obtained on operation Roller Coaster[4] when the standard geometrical deviation in the aerosol respiratory retention factor was $\approx$ 2.0.

We found that after a single intravenous or intratracheal injection of Po-210 to rabbits such major body organs as kidneys, liver, spleen and lungs contain various amounts of the radionuclide. This variation may be characterized by $\approx$ 2.5. With due correction for individual differences of animal species the result obtained may be interpreted as it was stated earlier thus assuming that if similar measurements could be made repeatedly on the some rabbit the lognormal distribution of the results would be obtained $\approx$ 2.0.

Summarising the cited above experimental data it may be concluded that all the cases being the result of relatively rapid physiological processes can be described (within the same individuum) by the lognormal fluctuation law with the standard geometrical deviation $\approx$ 2.0. This value β can be considered as basic for all other statistic estimations obtained by monitoring internal radiation exposure.

However it should be noted that the lognormal distribution

cannot be accepted directly as the main probability law for describing most parameters estimated in the course of various methods used for monitoring internal radiation exposure. Thus, we showed that even when the size distribution of radioactive airborn particles was fully suited by the mathematical equations of A. N. Kolmogorov's theory[6] it was necessary to account for limitation of the lognormal distribution from the side of large values. For other radiation parameters the use of the lognormal distribution may be considered valid for the range of 95÷99% of cases where it should be treated as a convenient and simple analitical expression.

As a result of the data discussed above we accept the following values of the standard geometrical deviations for primary processes, which are superposing into real situations discussed below:

β_1 =2.0 - retention in the respiratory organs;
β_2 =2.0 - fraction of the substance entering the organ;
β_3 =2.0 - elimination of the substance with urine and feci;
β_4 =1.6 - difference in AMAD of the inhaled airborn perticles;
β_5 =3.2 - difference in the local concentrations;
β_6 =3.6 - difference in the protective efficiency of respirators;
β_7 =7.0 ÷25 difference of the annual individual intake;
β_8 =5.0 - difference of the radioactive releases with account for accidental situations.

The choice of the last value is purely subjective assuming that the release which results in the median average for the day radioactive concentration being exceeded by 1000 times corresponds to 0.01% of cases.

3. Discussion

Practically any dosimetric control aimes at obtaining such result which could provide an unequivocal answer about the degree of individual risk, e. g. internal irradiation of man at a certain time moment. The result obtained by control is compared to the standard values. Basing on the data[7, 8] critical organs of the group (whole body, hemopoietic organs) the following limiting radiation dose values may be given:

1) annual permissible radiation dose for individuals from the population - 0.5 rem;
2) annual radiation dose for occupational workers who are not subjected to individual control - 1.5 rem;
3) annual permissible radiation dose for occupational workers - 5 rem;
4) permissible accidental radiation dose - 12 rem;
5) accidental radiation dose requiering subsequent medical examination - 25 rem;
6) accidental radiation dose which does not result in detectable immediate somatic effects - 75 rem;
7) sublethal radiation dose - 200 rem;
8) accidental radiation dose with a possible death in the absence of medical aid - 400 rad;
9) accidental radiation dose with survival of people only in case of intensive and immediate medical aid - 800 rad;
10) accidental radiation dose which permit survival in case of intensive and immediate medical aid - 1200÷1500 rad.

Basing on these limiting dose values we established 11 ranges, i. e. ranks of radiation hazard. The phylosophy for interpreting each of these ranks is different but now we are most in-

terested in the practical aspect of the problem, i. e. how effective the monitoring itself will be from the point of view of possible errors. As one of the specific methods for monitoring we may consider calculated prediction of radiation situation for design objectives.

Currently existing methods for monitoring external gamma-neutron radiation are usually fairly precise to provide unequivocal identification of the hazard rank by the result obtained. In the worst instance one may overestimate or underestimate the actual hazard rank not more than by 1. As for the results obtained by monitoring internal irradiation the situation is quite different even if to consider the optimum result equal to the average geometrical value of the lower and upper limits of the corresponding radiation dose range.

Table 5 gives conventional classification for the situations which occur at monitoring internal radiation exposure. The situations considered are characterized: by the corresponding standard geometrical deviation of the monitoring result from the possible real value; by the probability in per cent corresponding to the boundary values of a hazard rank; by the number of ranks comprising 99% of cases. The adjacent limiting radiation dose values listed above differ from each other not more than threefold. This maximum value is accepted for estimating the probability which corresponds to one hazard rank, i. e. the monitoring result differs from the corresponding range limits by $\sqrt{3}$ times. The number of ranks comprising 99% of cases was estimated relative to the result which corresponded to the 6th hazard rank. The resulting standard geometrical deviation was determined by the equation:

$$\beta = \exp\left\{\sqrt{\sum_k (\ln \beta_k)^2}\right\}$$

where β_k - standard geometrical deviations characterizing the primary processes cited at the beginning. The third column of Table 5 lists those β_k which were considered for the given situation.

Situation 11 (designed calculation) was estimated somewhat differently. It was supposed that the calculation was based on the average annual permissible concentration with the safety margin factor of 10. This calculated value evidently cannot be assigned to the 6th hazard rank but will be at the 1st one of the average annual value is taken for occupational workers. Therefore the probability in the last but one column was calculated for the ranks 1 and 2.

Of course, Table 5 is relatively limited. In addition the parameters for situation 11 were taken on the basis of subjective choice. However these data do not overestimate the values of the resulting difference.

The estimation of the hazard rank may be more complicated in a number of practical situations which were not considered here. For instance, there are other routes of intake in addition to inhalation. The errors associated with measuring techniques and a limited accuracy in description of metabolic processes were not accounted for also.

In this connection the estimates given in two last columns of Table 5 are very important. In neither of the situations considered the actual individual hazard of internal irradiation can be assigned unequivocally to one hazard rank. In a number of cases (situations 1, 2, 4, 7b, 10) the 99% range comprises 10-11 hazard ranks, i. e. the whole hazard scale. Thus, it is clear that a statistic estimate is an integral whole of the problem

for providing radiation safety of people. The authors deliberately neglected the fact that the original standard data included their own safety margin factors. These factors do not influence the quantitative estimate of the situation.

Conclusion

We believe that a statistic difference found in monitoring internal irradiation of people should be given a quantitative estimate of a wide usage. It will be possible only after international agreement on the main values. As the first step in this direction the authors suggest to introduce a concept of metabolism fluctuation into the characteristics of a standard man and to accept the standard geometrical deviation 2.0 for estimating radionuclide climination rate fluctuation.

Table 5

Different situations found in monitoring internal irradiation

No i	Situations	Primary standard geometrical deviations β_α	Resulting standard geometrical deviation β	Probability corresponding to the ranges of one hazard rank, %	The number of hazard ranks comprising 99% of cases (at the 6th ranks)
1	Estimate of the average annual content by a single measurement with a whole body counter for rapidly eliminated radionuclides	β_2, β_3	8.0	21	11
2	Monitoring a single intake by an individual sampler when the respirator of the "Lepestok" type is used	$\beta_1, \beta_2, \beta_3, \beta_4$	7.6	21	11
3	Monitoring a single intake by a stationary sampler when the respirator of the "Lepestok" type is used	$\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$	10.0	19	11
4	Monitoring a single intake by a stationary sampler in the absence of individual protective means	$\beta_1, \beta_2, \beta_4, \beta_5$	4.9	28	10 (N2+N11)
5	Monitoring a single intake by an individual sampler in the absence of individual protective means	$\beta_1, \beta_2, \beta_4$	3.0	38	7 (N3+N9)
6	Estimate of the single inhalation intake of radionuclide by several results of the complex bioassay	$\beta_2, \beta_3,$	2.7	42	7

No i	Situations	Primary standard geometrical deviations β_{st}	Resulting standard geometrical deviation β	Probability corresponding to the ranges of one hazard rank, %	The number of hazard ranks comprising 99% of cases (at the 6th ranks)
7	Estimate of the radionuclide content in the lungs by a timely single analysis of urine and feci	β_2,β_3 β_4	3.0	38	7 (N3÷N9)
8	Estimate of the radionuclide content in the body (except the lungs) by a single analysis of urine and feci:				
	a) at the time of measurement	β_2,β_3	2.7	42	7 (N3÷N9)
	b) on the average for a year	β_2,β_3,β_4	9.0	20	11
9	Monitoring a single intake by smears from the nosal orifices	β_2,β_4	2.3	49	5(N4÷N8)
10	Monitoring a single intake by smears from the nosal orifices combined with a selective individual sampler	β_2	2.0	57	5 (N4÷N8)
For example					
	Project of stationary aerosol protection based on the average annual permissible concentration (with averaged 10-fold safety margin)	β_1,β_2 β_3	8.3	99	2 (N1, 2)

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