A MATHEMATICAL FORMULATION OF RADIATION RISK

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INTRODUCTION

The purpose of radiation protection (RP) is self–evident but is not so easy to establish a target for it. The aim of RP has been set to prevent the so–called deterministic effects and also to suppress the stochastic effect to some acceptable limit. We have learnt through our experience of more than thirty years with this subject that under provision of the ordinary measures to achieve the latter aim, the former is usually automatically carried out.

Thus, the measure of the stochastic effect, radiation risk, is one of the most important two quantities in the causality relation, indispensable to radiation protection as a science. The quantity of cause is of course the radiation dose.

In this paper, issues of the definition and mathematical formulation of the radiation risk are discussed.

THE STATUS QUO

The term risk is widely used not only in the field of RP but also in many other fields. However, definition or significance of the word is not the same. In the field of RP, the word is often used to mean the probability of loosing an individual's life or mathematical expectation of the whole loss of lives in a population when it is used as the name of a quantity. While in some other fields including nuclear safety, risk is used as a synonym for the mathematical expectation of the magnitude of the undesirable consequence of event.

The risk defined as a synonym for the probability of incidence of lethal cancer, for example, is frequently understood as the rate of death due to cancer of an individual and expressed in the form of a percentage. Arithmetic treatment of this quantity we find unconvincing because total of these rates is fixed as one and the rate of carcinogenecis cannot be changed without in principle reshuffling other rates. Moreover, appearance of new death factors or increase in value of an existing other factor makes the value of the radiation risk decrease. When social desasters such as famine or AIDS occurs, the radiation risk decreases as result. Thus the risk of radiation defined with the probability is not suitable to discuss the risk of an individual.

ICRP introduced a concept of risk coefficient, risk per unit (effective) dose, which is used in the present system of radiation protection. However, as is well known, harmful stochastic effects of radiation on individuals depend on the age of exposure and their appearance accompanies stochastic "latent periods." On the other hand, risk of an exposure at time t_0 in the past ($t_0 < 0$) for the time passed is deterministic and for the future is stochastic. The function of life existence can have values of either one or zero for the past, while it can have a value of a positive real number between zero and one. Thus, even with the same exposure, evaluated risk depends on the time of evaluation even if the character of the population is stable. Therefore, the ICRP risk coefficient is not so useful for the purpose of discussing the risk to an individual.

Shortening of the mean residual life span of a population is often used as an expression of risk. This is a mathematical expectation. Extinction or depletion of the species is also important as a harmful effect of radiation on human beings and this quantity is useful for considering this problem. However, it is not suitable for discussing an individual's risks, because its value depends on the size of the population.

A MATHEMATICAL FORMULATION OF RADIATION RISK

In order to overcome the above-stated difficulties of the present situation, a reformulation of the radiation risk is tried on the premise of linear proportionality between the dose and the effect and on the newly specified concept of radiation risk.

We define the risk accompanying an act of receiving some amount of radiation dose as the expected value of shortening of the residual life span of a hypothetical standard man (or woman) at the time of exposure. The standard man (or woman) is a hypothctical individual characterized by the statistical data, i.e., the mortality rate as a function of age, $\lambda(\tau)$, with a population of interest. The data is obtained from a national census or such. His or her residual life span at age τ, $\rho(\tau)$, is calculated with the mortality function $\lambda(\tau)$ by the following relations;

$$-dN(\tau)/d\tau = \lambda(\tau)N(\tau) \quad (1)$$

$$\rho(\tau_0) = \int_{\tau_0}^{\infty} \tau\, \sigma(\tau_0, \tau)\, d\tau - \tau_0 \quad (2)$$

where τ_0 is the present time (age) and $\sigma(\tau_0, \tau)$ is the probabolity of maintaining life of an individual at age τ,

$$\sigma(\tau_0, \tau) = 1 \qquad \text{for } \tau \leq \tau_0$$

$$\sigma(\tau_0, \tau) = N(\tau)/N(\tau_0) \qquad \text{for } \tau > \tau_0 \quad (3)$$

$N(\tau)$ is the population of the standard men (or women) of age τ.

Now the risk brought by unit amount of instantaneous dose is expressed as follows:

$$r(\tau, \tau') = \rho(\tau) - \rho'(\tau, \tau') \quad (4)$$

where $\rho'(\tau, \tau')$ is the new function or curve of the expected residual life span for an instantaneous exposure of unit amount of dose at the time (age) of τ'. The function $\rho'(\tau, \tau')$ can be evaluated if the data on the distortion of the function $\lambda(\tau)$ for an instantaneous exposure of unit amount of dose at time of τ' is available.

$$\dot{\varepsilon}(\tau) = \delta(\tau'), \quad (5)$$

where $\delta(\tau)$ is the Dirac's delta function and $\varepsilon(\tau)$ is a time spectrum of the dose,

$$\dot{\varepsilon}(\tau) \equiv dE(\tau)/d\tau$$

Thus the risk (for the future only by nature) evaluated at the time τ for the past exposure $\dot{\varepsilon}(\tau')$ is expressed as

$$R(\tau) = \int_{\tau}^{\infty} r(\tau, \tau')\, \varepsilon(\tau')\, d\tau' \quad (6)$$

Here, the risk or expected shortening in residual life span is proportional to the dose is assumed. That for the past is zero due to the fact of survival at the time of evaluation.

CONCLUSION

Risk for radiation exposure is defined as the expectation of the shortening of the residual life span. With this definition, the concept of risk is only for future life, since the fact of maintaining life at the time of evaluation negated the possibility of losing the life in the past.

A mathematical formulation is shown under the premise of the linear proportional relation between the dose and the end effect (newly defined risk) and on the new concept of "Standard Man/Woman" whose characteristic on mortality is the same of the population of interest.

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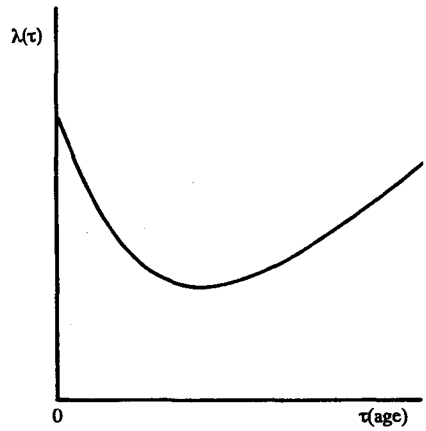

Fig.1 Mortality Function

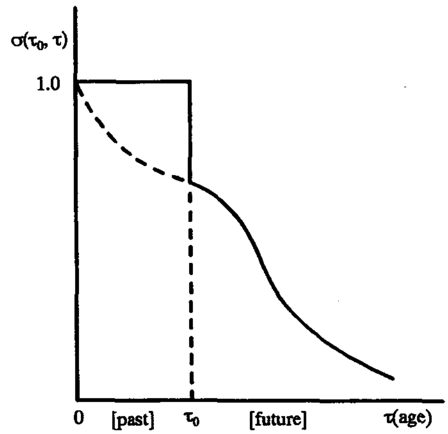

Fig.2 Existence Function

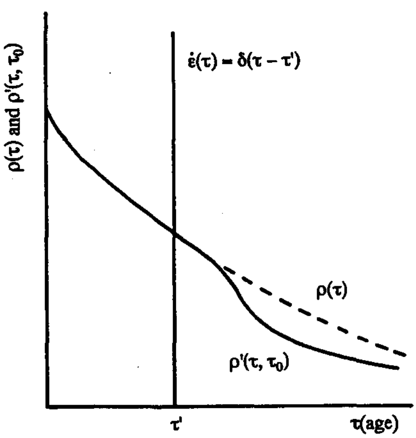

Fig.3 Expected Residual Life Span