The evaluation of multi-element personal dosemeters using the linear programming method

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Introduction

Multi-element dosemeters are frequently used in individual monitoring. Each element can be regarded as an individual dosemeter with its own individual dose measurement value. In general, the individual dose values of one dosemeter vary according to the exposure conditions, i. e. the energy and angle of incidence of the radiation. The (final) dose measurement value of the personal dosemeter is calculated from the individual dose values by means of an evaluation algorithm. The best possible dose value, i. e. that of the smallest systematic (type B) uncertainty if the exposure conditions are changed in the dosemeter's rated range of use, is obtained by the method of linear programming.

Mathematical basis

The mathematical problem of determining the measured dose value H of the entire dosemeter from the individual dose values of each detector element $\{D_k, k = 1, \cdots, m\}$ has been described by Bermann and Chanourdie [1] in the form of a set of linear algebraic equations:

$$D_k = \sum_{i=1}^{n} a_{i,k} \cdot H_i \qquad k = 1, \cdots, m \tag{1}$$

$$H = \sum_{i=1}^{n} H_i \tag{2}$$

Here n denotes the number of the different radiation fields used for the calibration, and m denotes the number of the individual detector elements in the dosemeter. In the evaluation of the dose value H, m individual dose values $\{D_k, k = 1, \cdots, m\}$ are therefore measured. The response matrix $a_{i,k}$ contains the response of the m individual detector elements of the dosemeter to the n different radiation fields used for calibration. In the evaluation of H it is assumed that the dosemeter has been exposed to a radiation field which can be described as a linear combination of these n calibration fields with dose contributions H_i, see eq. (2).

The solution of a set of linear algebraic equations of the form of eq. (1) and (2), i. e. here the determination of a set of dose contributions $\{H_i, i = 1, \cdots, n\}$, is completely covered by the theory of linear programming and was described in [2] including the following two problems:

- Frequently the set of equations (1) cannot be solved for a set of measured individual dose values.
- If the set of equations (1) can be solved, usually the solution is not a unique one. In this case, the dose values H, see eq. (2), form an interval:

  $$H_{min} \leq H \leq H_{max} \tag{3}$$

  with all values of H within the interval being solutions of the set of equations (1) and (2). This problem has been described in [3].

At present, in radiation protection the dose evaluation algorithms listed in table 1 are applied to determine the dose value H from the individual dose values $\{D_k, k = 1, \cdots, m\}$. Each of these evaluation algorithms can be interpreted as an attempt at finding a solution of the set of equations (1) and (2). The algorithms differ with respect to the information made available concerning the uncertainty of the dose value H and concerning the dose contributions $\{H_i, i = 1, \cdots, n\}$ to the dose value H, i. e. the spectrum of the radiation field.

***Table 1.** Additional information resulting from the various dose evaluation algorithms.*

Evaluation method	Additional information about:	
	uncertainty	spectrum
(A) Methods using ratios of the individual values	No	Yes
(B) Linear combination method	Yes	No
(C) Linear programming method	Yes	Yes

At least in Germany, above all the algorithm (A) is applied at present. To support the dose evaluation by algorithms (B) and (C) a program named »LINOP« and the corresponding manual were developed [4]. The program allows both the determination of the optimum coefficients for the linear combination method and the dose evaluations by linear programming using the simplex method to be carried out in the routine monitoring of a dosimetry service.

In the following, the evaluation algorithms mentioned previously are described in more detail. The results of a test of »LINOP« by corresponding measurements are presented.

Methods using ratios of the individual values

By the methods using ratios of the individual values (A), one single dose value compatible with the set of equations (1) and (2) and the corresponding spectrum are determined. The set of equations is not completely solved, i. e. the interval $[H_{min}, H_{max}]$ is not determined. The uncertainty of the measured dose value must therefore be assumed to be of the size found in performance tests carried out with well-defined radiation fields and well-defined irradiation conditions. The actual uncertainty cannot, however, be determined by these methods.

Linear combination method

By the linear combination method (B), the set of equations (1) and (2) is solved neglecting the information concerning the spectrum, which are contained in the individual dose values:

$$H = \sum_{k=1}^{m} \sigma_k \cdot D_k \qquad (4)$$

The optimum coefficients σ_k minimizing the uncertainty of measurement of this method are obtained by linear programming [2, 4] using eq. (1) and (2) together with the minimization of the maximum uncertainty of measurement of all calibration measurements. The uncertainty of measurement to be specified in the evaluation of the dose value is independent of the individual dose values $\{D_k, k = 1, \cdots, m\}$ of a single measurement, but is equal to the maximum interval $[H_{min}, H_{max}]$ resulting from all individual dose values of all calibration measurements. Neglection of the information about the spectrum therefore leads to an increasing uncertainty of measurement.

Linear programming method

For every measurement the method of linear programming (C) [2, 4] furnishes the set of dose values compatible with the set of equations (1) and (2), i. e. the dose interval $[H_{min}, H_{max}]$ depending on the individual dose values $\{D_k, k = 1, \cdots, m\}$ of the respective measurement. The spectra corresponding to H_{min} and H_{max} can be specified. This method is applicable to every measurand and to every multi-element dosemeter and, in every case, yields the minimum uncertainty of measurement.

The results of the »LINOP« program were verified by measurements with a four-element laser TLD dosemeter [5] (see table 2). The response matrix $a_{i,k}$ of this TL-dosemeter consists of the responses of the four single elements to X-ray radiation of the ISO narrow spectrum series (N-30 to N-300) and to gamma radiation of ^{137}Cs and ^{60}Co at angles of 0°, 30°, 45° and 60° on an ISO water slab phantom. The measured value M_p of the personal dose equivalent is the geometric mean of H_{min} and H_{max}. The maximum possible dose interval $[H_{min}, H_{max}]$ was calculated by »LINOP«. The calculation of the maximum dose interval not only furnishes the corresponding calculated individual dose values D_k^c of the four single elements but also the irradiation conditions, i. e. the dose contributions H_i of the radiation qualities Q_r at the angles of incidence α, which lead to these individual dose values D_k^c. These irradiation conditions are calculated for $H = H_{min}$ and $H = H_{max}$, H being the dose to which the dosemeter is exposed. Irradiation of the dosemeter in a radiation field satisfying these calculated irradiation conditions and subsequent evaluation of the dosemeter lead to the measured individual dose values D_k^m which are slightly different from the calculated dose values D_k^c which then result in a slightly different dose interval $[H_{min}, H_{max}]$.

The comparison of the calculated and measured individual dose values and the resulting dose interval listed in table 2 yields good agreement between calculation and experiment. The calculated and measured data differ only slightly within the uncertainty which can be achieved with the TL-detectors used.

Table 2. *Comparison between calculated irradiation data and the corresponding measured irradiation data, both obtained by linear programming. For the symbols, see text.*

	Irradiation conditions					Individual dose values in mSv				Dose in mSv			
$H=$	Q_r	α	H_i in mSv	H in mSv		D_1	D_2	D_3	D_4	H_{min}	H_{max}	M_p	$\frac{M_p}{H}$
H_{min}	N-30	45°	0,08	2,00	calc.	6,70	2,29	4,77	6,50	2,00	2,69	2,32	1,16
	N-150	60°	1,92		meas.	6,72	2,15	4,48	6,52	1,90	2,56	2,21	1,11
H_{max}	^{60}Co	0°	2,21	2,69	calc.	6,70	2,29	4,77	6,50	2,00	2,69	2,32	0,86
	N-60	30°	0,48		meas.	6,60	2,30	4,66	6,57	2,08	2,73	2,38	0,88

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