Disaster Management and Control: A Stochastic Process Approach

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ABSTRACT

Uncertainty is an important characteristic in many real-life settings. Crises and disasters are two of them. They bring about conditions that could lead to undesirable economics or unacceptable HSE state of affairs. Disaster management stands for processes and procedures aimed at dealing with a disaster in a manner that minimizes damage and allows the affected population to recover quickly. Critical industries, such as nuclear power generation plants, are aware of the need for efficient disaster preparedness and response.

In disaster settings, the exact times and magnitudes of the events cannot be predicted with certainty. They are probabilistic events, and require corrective intervention measures. Intervention requires the optimal allocation of scarce resources. Therefore, researchers recognize the need to establish a cost-effective (efficient) management and control of disaster events.

In general, published literature in this area seeks to provide more accurate and/or better technological infrastructure of systems to support disaster recovery efforts. However, there is a need to examine key features of stochastic systems of natural events that may affect the decision-making process in nuclear and radiological disaster monitoring, control and recovery, in order to release pressure on operators in NPP control rooms.

This paper treats the inventory of *direct* losses of a disaster as a stochastic system (human casualties, property destroyed, damaged and recoverable material and equipment). It proposes the Queuing Theory (a Markovian birth-and-death stochastic process) to model the inventory system of human casualties. This is a stochastic method, used extensively in Operations Research to model queuing and inventory systems, health care systems, inventory management, quality control and inspection, and population dynamics.

Here, the Queuing Theory is used as a model for the analysis, management and control of disaster recovery efforts. Medical treatment centers are assumed to receive casualties arriving and serviced in a Poisson probability distribution. That is, Poisson distribution is used to analyze a system of medical relief centers (disaster outcomes) as a queuing system in which both arrivals and service of casualties are assumed completely random in time, with parameters:

- λ: mean rate of arrival; equal to 1/E[Inter-arrival-Time], where E[.] denotes the expectation operator.
- μ: mean medical service rate; equal to 1/E[Service-Time].
- $\rho = \lambda/\mu$ for single server queues: utilization of the server; also the probability that the server is busy or the probability that someone is being served.
- c: number of medical servers.

The design of a queuing system to manage and control the inventory of casualties is optimal when a steady-state prevails. If the number c of required medical servers is to be determined, the procedure starts with determining the smallest integer c such that the "service center utilization factor"= <1 and to study the resulting values of the corresponding "measures of effectiveness" until a specific measure (such as the "waiting time") is obtained that is acceptable by the Disaster Management Center or Disaster Monitoring Authority from an economic or humanitarian disaster recovery point of view.

Keywords: Disaster, Management, Queueing Theory, Markovian Theory

1. INTRODUCTION

Uncertainty is an important characteristic in many real-life settings. In industrial accidents, for example, a plant can run into circumstances that impede their safe performance. In industrial accidents, the exact times and magnitudes of the events cannot be predicted with certainty. Such events require rapid intervention measures, which require the optimal allocation and deployment of scarce resources. Hence, there is a need to examine key features of stochastic systems of natural events that may affect the decision-making process

in nuclear and radiological disaster monitoring, control and recovery, in order to release pressure on operators in NPP control rooms **[1-5]**.
This paper treats the inventory of *direct* losses of a disaster as a stochastic system (human casualties). It proposes the Queuing Theory (a Markovian birth-and-death stochastic process) to model the inventory system of disaster events or outcomes. This is a stochastic method, used extensively in Operations Research to model queuing and inventory systems, health care systems, inventory management, maintenance and availability, logistics, and population dynamics **[1-14]**.

Here, the Queuing Theory is used as a model for the analysis, management and control of disaster recovery efforts. Service centers are assumed to receive customers (i.e. disaster casualties) arriving and serviced in a Poisson probability distribution. That is, Poisson distribution is used to analyze a system of relief service centers (disaster outcomes) as a queuing system in which both arrivals and service of customers (i.e. casualties) are assumed completely random in time, with parameters:

- λ: mean rate of arrival; equal to 1/E[Inter-arrival-Time], where E[.] denotes the expectation operator.
- μ: mean service rate; equal to 1/E[Service-Time].
- $\rho = \lambda/\mu$ for single server queues: utilization of the server; also the probability that the server is busy or the probability that someone is being served.
- c: number of service centers.

Therefore, the need is recognized to establish a cost-effective (efficient) management and control of the situation.

2. FORMULATION OF THE MODEL

We treat the "disaster recovery" efforts as a "Queuing System". A queuing system is a population growth process, where a customer (in this case a "casualty") arrives at a medical help facility, wait in a queue if all servers are busy or receive medical help and finally leave the facility, Fig. 1.

Denote by $\{N(t); t \geq 0\}$ the number of casualties arriving at the medical help at time $t \geq 0$. If $\{N(t); t \geq 0\}$ is a "Queuing System", its realizations are step functions that start at N(0) and have upward jumps of size 1 when a casualty arrives for medical care and downward jumps of size 1 when a casualty leaves.

2.1 Assumptions

The following simplifying assumptions are made:

Queueing System: We assume an initial number N(0) of casualties in the medical help system at time t_0. This medical help system is a *Poisson birth-death population growth process* with a population consisting of N(t) casualties either waiting for medical help or currently being helped at time t. A "*birth*" occurs when a casualty arrives and *enters* the system. A "*death*" occurs when a casualty gets the needed help and *leaves* the system. The state of the system, at time t, is the number N(t) of casualties still under medical care in the system.

Queue Characteristics: Using the *Kendall-Lee notation*, we specify the characteristics of the Queueing System by 5 parameters: *v/w/x/y/z*, where:

v refers to the pattern in which casualties arrive at the medical facility for help (known as "arrival pattern").

w refers to the medical intervention pattern (known as "service pattern").

x refers to the number of intervention staff. We assume that no casualty waits while an intervention staff is idle.

y refers to the medical facility system "capacity"; refers to the maximum number of casualties either being attended to or waiting in the queue.

z refers to the order in which casualties are attended to.

2.2 Derivation of the Model

Using the *Kendall-Lee notation* (v/w/x/y/z) for the characteristics of a queue, we first consider an M/Mc/K queueing system for the management of disaster casualties, where:

M specifies the arrival pattern as a Markovian (i.e. stochastic) birth process, with "average arrival rate" λ. We assume the inter-arrival time, 1/ λ, to follow the exponential probability distribution; P ($T \leq t$) = 1-$e^{-\lambda t}$; $t \geq 0$.

M specifies the service pattern as a Markovian (i.e. stochastic) death process, with "average service rate" μ. We assume the service time, 1/μ, to also follow the exponential probability distribution; P ($T \leq t$) = 1-$e^{-\mu t}$; $t \geq 0$.

c is the number of intervention staff in the medical facility available to attend to arriving casualties ; c is a finite integer c > 0.

K specifies the "capacity" of the queueing system; limited integer; equal to a finite integer K>0.

The queue discipline, *z* in the *Kendall-Lee* notation, is assumed here to be a "first-in-first-out" (FIFO) discipline, i.e. service is in order of arrival.

Intuitively, K>c, since the maximum number of casualties must be as large as the number of medical staff available to attend them.

The "state" of the queueing system at time *t*, is given by the state probability $P_n(t)$ that the system has *n* casualties either waiting for attendance or being attended to.

The above assumptions yield the following arrival rate λ_n and service rate μ_n **[6, 9, 13, 14]**

$$\lambda_n = \begin{Bmatrix} \lambda \ \ for\ 0 \leq n < K \\ 0 \ \ for\ n \geq K \end{Bmatrix} \quad (1)$$

$$\mu_n = \begin{Bmatrix} n\mu \ \ for\ 0 \leq n \leq c \\ c\mu \ \ for\ n > c \end{Bmatrix} \quad (2)$$

Define the medical facility "utilization factor", ρ, as the expected number of casualties arriving per mean service time, μ_n:

$$\rho \equiv \frac{\lambda_n}{\mu_n} \quad (3)$$

In other applications, ρ is also called the "traffic intensity" of the queue.

The queue can reach a steady state if, and only if, $\rho \leq 1$. If $\rho > 1$, casualties arrive at a higher rate than the medical facility can service them. That is, the queue would grow without control and a steady-state cannot prevail.

For this finite capacity system (M/M/c/K), steady state exists for all $\rho = \frac{\lambda}{c\mu}$.

The *forward Kolmogorov equations* **[6, 7, 9, 13]** give the state probabilities for the generalized Markovian birth-death process:

$$\frac{dp_n(t)}{dt} = -(\lambda_n + \mu_n)p_n(t) + \mu_{n+1}p_{n+1}(t) + \lambda_{n-1}p_{n-1}(t) \qquad \textit{for } n > 0 \qquad (4a)$$

And

$$\frac{dp_0(t)}{dt} = -\lambda_0 p_0(t) + \mu_1 p_1(t) \qquad \textit{for } n=0 \qquad (4b)$$

The steady-state probabilities are derived by setting the steady-state condition $dp_n/dt=0$ in the above equations, which yields **[6, 9, 13, 14]**:

$$p_n = \frac{\lambda_{n-1}}{\mu_n} p_{n-1} \qquad (5)$$

Or

$$p_n = \frac{\lambda_{n-1}}{\mu_n}\frac{\lambda_{n-2}}{\mu_{n-1}} \dots\dots \frac{\lambda_0}{\mu_1} p_0 \qquad (6)$$

Where,

$$p_0 = \left[\frac{c^c}{c!}(K-c) + \sum_{n=0}^{c}\frac{c^n}{n!}\right]^{-1} \quad \text{for } \rho = 1 \qquad (7a)$$

And

$$p_0 = \left[\frac{c\rho^{c+1}(1-\rho^{k-c})}{c!(1-\rho)}(K-c) + \sum_{n=0}^{c}\frac{(c\rho)^n}{n!}\right]^{-1} \quad \text{for } \rho \neq 1 \qquad (7b)$$

And

$$p_n = \begin{cases} \frac{(c\rho)^n}{n!}p_0 & for\ 1 \leq n \leq c \\ \frac{c^c\rho^n}{c!}p_0 & for\ c < n \leq K \\ 0 & for\ n > K \end{cases} \qquad (8)$$

2.3 Measures of Service Effectiveness

The measures of effectiveness of response actions taken during an emergency response to save lives and/or attend to casualties are given in terms of **[6, 8, 14]**:

1. The average number of casualties in the queueing system (denoted by L),
2. The average length of the queue (denoted by L_q),

3. The average time a casualty spends in the system (Queue + Emergency Medical Intervention); denoted by *W*,
4. The average time a casualty spends in the queue waiting for medical service (denoted by W_q),
5. The probability that a casualty spends at least t units of time in the system (denoted by $W_{(t)}$), and
6. The probability that a casualty spends at least t units of time in the queue waiting for service (denoted by $W_{q(t)}$).

The first four measures (L, L_q, W, and W_q) are related by the following logic **[14]**:
Expected waiting time in the System = Expected waiting time in the queue + Expected medical service time. Hence,

$$W = W_q + \frac{1}{\mu} \qquad (9)$$

where μ is the "average service rate", defined above.
Define $\bar{\lambda}$ as the *effective arrival rate*, which refers to only those casualties that actually join the system (i.e. when the system capacity *K* is not reached).

Multiplying the equation (9) $W = W_q + \frac{1}{\mu}$ by $\bar{\lambda}$, we get, $L = L_q + \frac{\bar{\lambda}}{\mu}$

Hence,

$$\bar{\lambda} = \lambda\,(1 - p_K) \qquad (10)$$

For the M/M/c/K queueing system in hand, the measures of effectiveness of emergency response to save lives and/or attend to casualties are given in terms of L_q, W_q, *W* and *L* as follows **[6, 13]**:

$$L_q = \frac{c\rho^{c+1}}{c!(1-\rho)^2}\left[\left(1-\rho^{k-c}\right) - (1-\rho)(K-c)\rho^{k-c}\right] p_0 \qquad (11)$$

$$W_q = \frac{L_q}{\bar{\lambda}} \qquad (12)$$

$$W = W_q + \frac{1}{\mu} \qquad (13)$$

$$L = \bar{\lambda}\, W \qquad (14)$$

Equations (11) and (13) are called *Little's Formulas*.

The calculation of the measures of effectiveness is carried out in the following sequence:

$$p_n \longrightarrow L = \sum_{n=0}^{c} n p_n \longrightarrow W = \frac{L}{\bar{\lambda}} \longrightarrow W_q = -\frac{1}{\mu} \longrightarrow L_q = \bar{\lambda} W_q$$

The Case of a Single-Queue, Single-Server System

A special, and simpler, case of the M/M/c/K queueing system is obtained when c=1. The M/M/1/K queueing system has the following governing equations **[6, 7, 9, 13, 14]:**

$$\lambda_n = \begin{Bmatrix} \lambda \;\; for\; 0 \le n < K \\ 0 \;\; for\; n \ge K \end{Bmatrix} \qquad (15)$$

$$p_n = \begin{cases} \frac{(c\rho)^n}{n!} p_0 & for\ 1 \leq\ n\ \leq c \\ \frac{c^c \rho^n}{c!} p_0 & for\ c <\ n\ \leq K \\ 0 & for\ n > K \end{cases} \quad (16)$$

$$L = \begin{Bmatrix} \frac{\rho}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}} & for\ \rho \neq 1 \\ \frac{K}{2} & for\ \rho = 1 \end{Bmatrix} \quad (17)$$

With W, W_q, L_q and $\bar{\lambda}$ as for the M/M/c/K queueing system.

The Case of Infinite-capacity M/M/1 System

If the restriction of having a limited capacity is lifted, we get an infinite-capacity system, denoted M/M/1.

Because of the infinite capacity, $\bar{\lambda} = \lambda$, and the measures of effectiveness become:

$$L = \frac{\rho}{1-\rho} \quad (18)$$

$$L_q = \frac{\rho^2}{1-\rho} \quad (19)$$

$$W = \frac{1}{\mu-\lambda} \quad (20)$$

$$W_q = \frac{\rho}{\lambda-\rho} \quad (21)$$

3. Balking and Reneging

3.1 Balking

Balking refers to the situation when a casualty arrives but not allowed to join the queue of the emergency service facility because the queue is too long, or because the casualty is in a state that requires different treatment. Balking is, therefore, state-dependent. It is modelled mathematically by a *Balking Probability Function B(n)*, which is the probability that a casualty arrives but will not join the queue for the two reasons mentioned above. The Probability that the casualty will join the queue is therefore = *1- B(n)*.
If the arrival rate λ is state-independent, the expected effective arrival rate into the emergency service facility is then, λ_n = [*1- B(n)*]λ which is a state-dependent. Note that *B(0) = 0.*

3.2 Reneging

Reneging refers to the situation where a casualty join the queue but then removed from it, either because the waiting time has become too long, or because of a change in service priorities. Reneging is therefore a state-dependent and acts to increase the mean service rate, μ. Reneging is modelled by defining a *Reneging Probability Function*, *R(n)*, as:

$$R(n) = \lim_{\Delta t \to 0} P\,\{a\ casualty\ will\ be\ removed\ in\ time\ \Delta t \mid n\ casualties\ in\ the\ queue\,\}/\Delta t$$

And adding it to the mean service rate of the emergency center. The effective service rate μ_n becomes:

$\mu_n = \mu + R(n)$. Note that $R(0) = R(1) = 0$

4. CONCLUSION

A stochastic process approach has been used to model the problem of disaster management and control of the inventory of human casualties. The disaster casualty system is treated as a "queueing system", and hence analyzed by the principles of queueing theory. We defined $p_n(t)$ as the probability of *n* casualties arrive at the emergency medical care center during time interval *t*. We assumed that the inter-arrival times and the service times both follow the exponential distribution $f(t) = \alpha e^{-\alpha t}$, where α is the rate at which casualties arrive at the care center. The distribution of *n* (arrivals or departures) during t is assumed to be a *Poisson Probability Distribution*, $p_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$, *n=0,1,2,3,…………*

The design of the queueing system to manage and control the inventory of the disaster casualties is *optimal* when a *steady-state* prevails. If the number *c* of *service stations* is to be determined, the procedure starts with determining the smallest integer *c* such that the "utilization factor" $\rho = \frac{\lambda}{c\mu} < 1$ and to study the resulting values of the corresponding "measures of effectiveness" until a specific measure (such as the "waiting time") is obtained that is acceptable to the Emergency Authorities.

The specific choice of the number *c* depends on what the Emergency Authorities consider as acceptable for that stage of disaster recovery.

Because of the unique "memoryless" property of the exponential distribution $f(t) = \alpha e^{-\alpha t}$ governing the inter-arrival times and the service times,, this unique property also shows that the Poisson process described by $p_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$, *n=0,1,2,3,………* is completely random, and that the probability distribution of *n* events (births or deaths) occurring during time interval t is a Poisson distribution.

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Figure 1: Components of an Emergency Queueing System